



# Assimilating precipitation observations into HWRF while avoiding the pitfalls of microphysical representations

Ziad Haddad, Svetla Hristova-Veleva, **Jeffrey Steward** and Tomi Vukicevic

[zhaddad@jifresse.ucla.edu](mailto:zhaddad@jifresse.ucla.edu)



Say we want to reconcile  $\mu$ wave brightness temperatures  $T_1, \dots, T_9$  measured over a horizontal pixel  $(\text{lon}_0, \text{lat}_0)$  with the model forecast values  $x_1, \dots, x_{504}$  for the model state variables:

Will need to find  $a_1, \dots, a_{504}$  that bridge the gap, i.e. which minimize

$$(a - x)^t B (a - x) + (H(a) - T)^t R (H(a) - T)$$

where “ $H(a)$ ” is the forward observation operator expressing how the brightness temperatures depend on the variables.

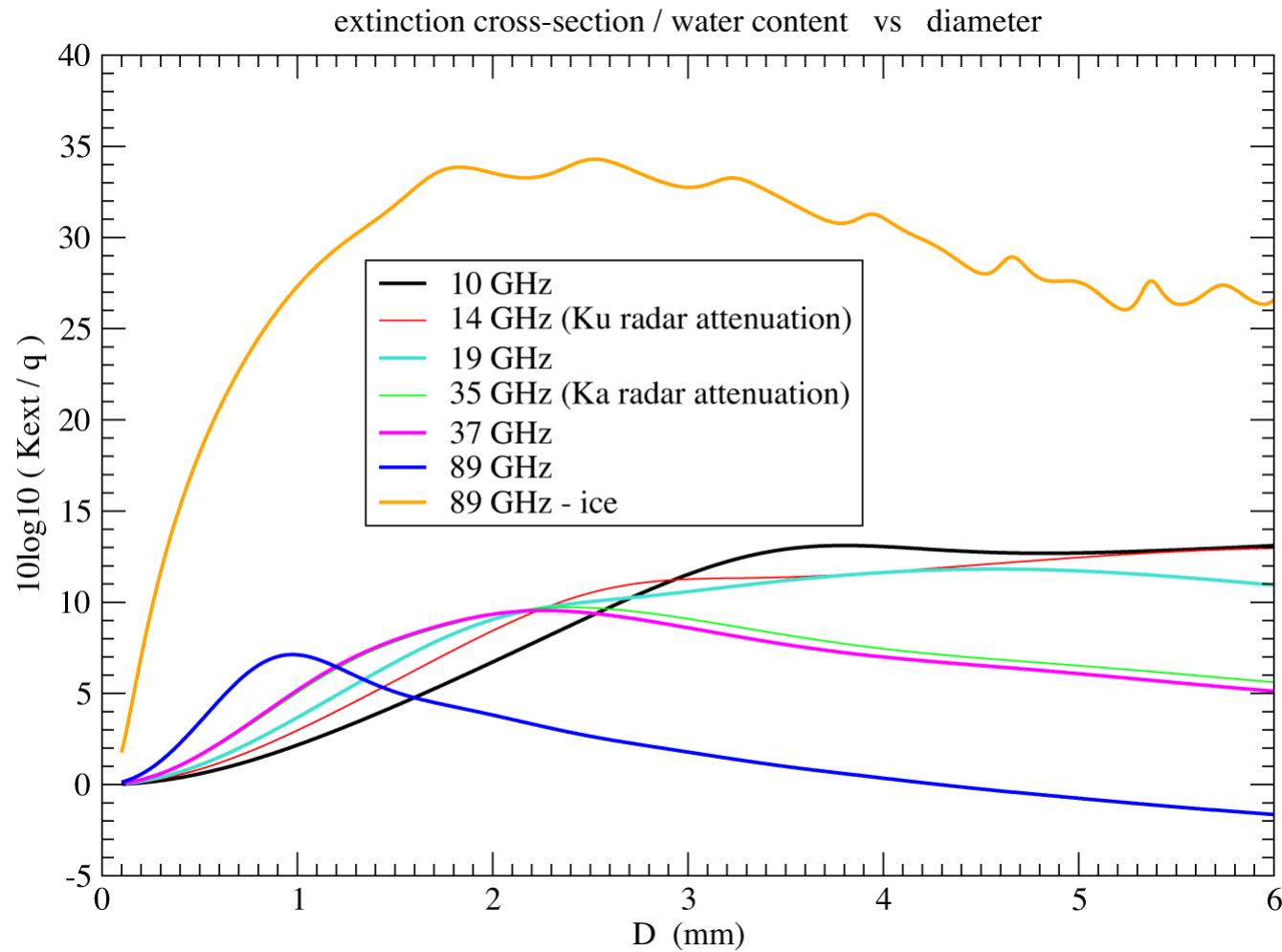
Problems:

- 1)  $H(a) = H(a; b)$ , with  $b$  crucial for  $H$  but not tracked by the model
- 2)  $T$  most sensitive to subset of variables  $a'$  whose correlation with remaining ones is not well quantified or well represented in  $B$
- 3) Model representation of  $a'$  and  $b$  not very realistic
- 4)  $H$  very non-linear  $\Rightarrow$  need care in representing dependence of  $H$  (and of its gradient) on  $a'$

## “unrealistic realism” in hydrometeor size distributions:

The size of the hydrometeors is very important to correctly interpret the microwave observations:

Crucial to know how the water content values  $q$  ( $\text{g}/\text{m}^3$ ) are distributed in size ...



⇒ Assume closed-form diameter distributions (e.g. exponential or  $\Gamma$ )

“unrealistic realism” in hydrometeor size distributions:

⇒ Assume closed-form diameter distributions (e.g. exponential or  $\Gamma$ ):

$$N(D) = N_0 D^\mu e^{-\Lambda D}$$

Let's try to interpret the parameters in terms of physically meaningful quantities:

$$D_m = \frac{\int D D^3 N(D) dD}{\int D^3 N(D) dD} = \frac{\mu + 4}{\Lambda}$$

$$q = \int \frac{4}{3} \pi \left( \frac{D}{2} \right)^3 \rho N(D) dD = \frac{\pi}{6} \frac{\rho \Gamma(\mu + 1)}{(\mu + 4)^{\mu + 1}} D_m^{\mu + 1} N_0$$

$$\sigma_m = \sqrt{\frac{\int (D - D_m)^2 D^3 N(D) dD}{\int D^3 N(D) dD}} = \frac{D_m}{\sqrt{\mu + 4}}$$

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In the models, typically assume  $N_0$  constant and  $\mu = 0$ .

What that implies is:

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$$= \frac{\pi \rho}{24} N_0 D_m$$

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$$D_m = \frac{24}{\pi\rho N_0} q$$

In particular,

- $D_m/q = \text{constant}$ , and
- $\max(D_m)/\min(D_m) = \max(q)/\min(q)$

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But  $3.5 \text{ mm} / 0.5 \text{ mm} \neq \max(R^{0.9})/\min(R^{0.9}) \approx 100 \text{ mm/hr} / 0.1 \text{ mm hr}$

“unrealistic realism” in hydrometeor size distributions:

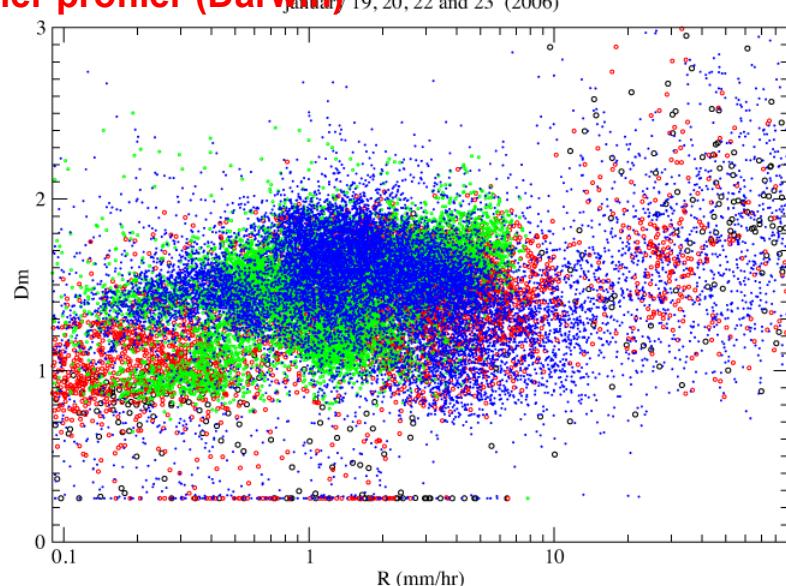
⇒ In fact, hydrometeor data suggests  $D_m \sim q^{0.17} \pm \text{white noise}$

$D_m \sim q^{0.17}$  behavior in profiler data

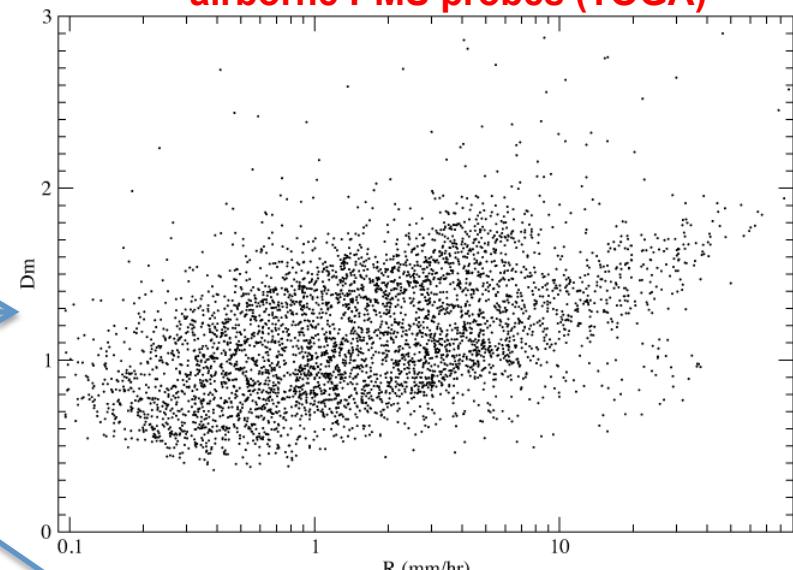
is consistent with TOGA-COARE

and Kwajex data ...

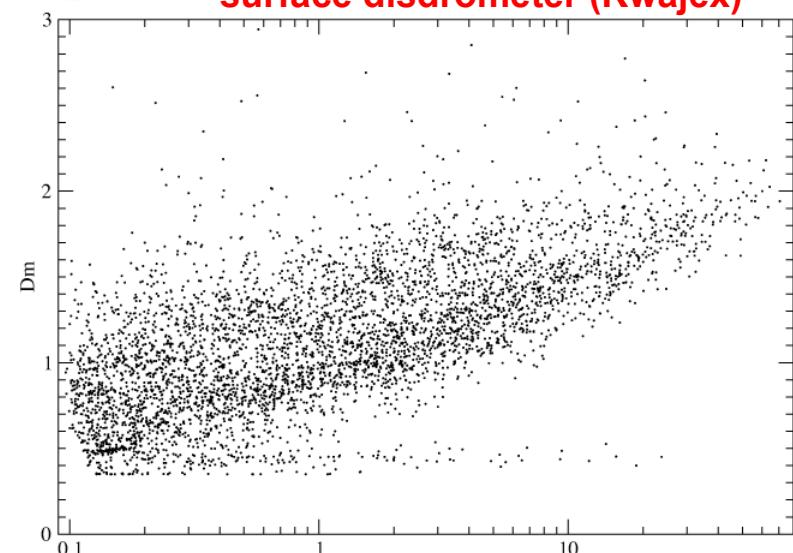
Doppler profiler (Darwin)



airborne PMS probes (TOGA)



surface disdrometer (Kwajex)



“unrealistic realism” in hydrometeor size distributions:

⇒ Assume closed-form diameter distributions (e.g. exponential or  $\Gamma$ ):

$$N(D) = N_0 \ D^\mu \ e^{-\Lambda D}$$

Fixing  $\Lambda$  is at least as problematic:

$$D_m = \frac{\int D \ D^3 N(D) \ dD}{\int D^3 N(D) \ dD} = \frac{\mu + 4}{\Lambda}$$

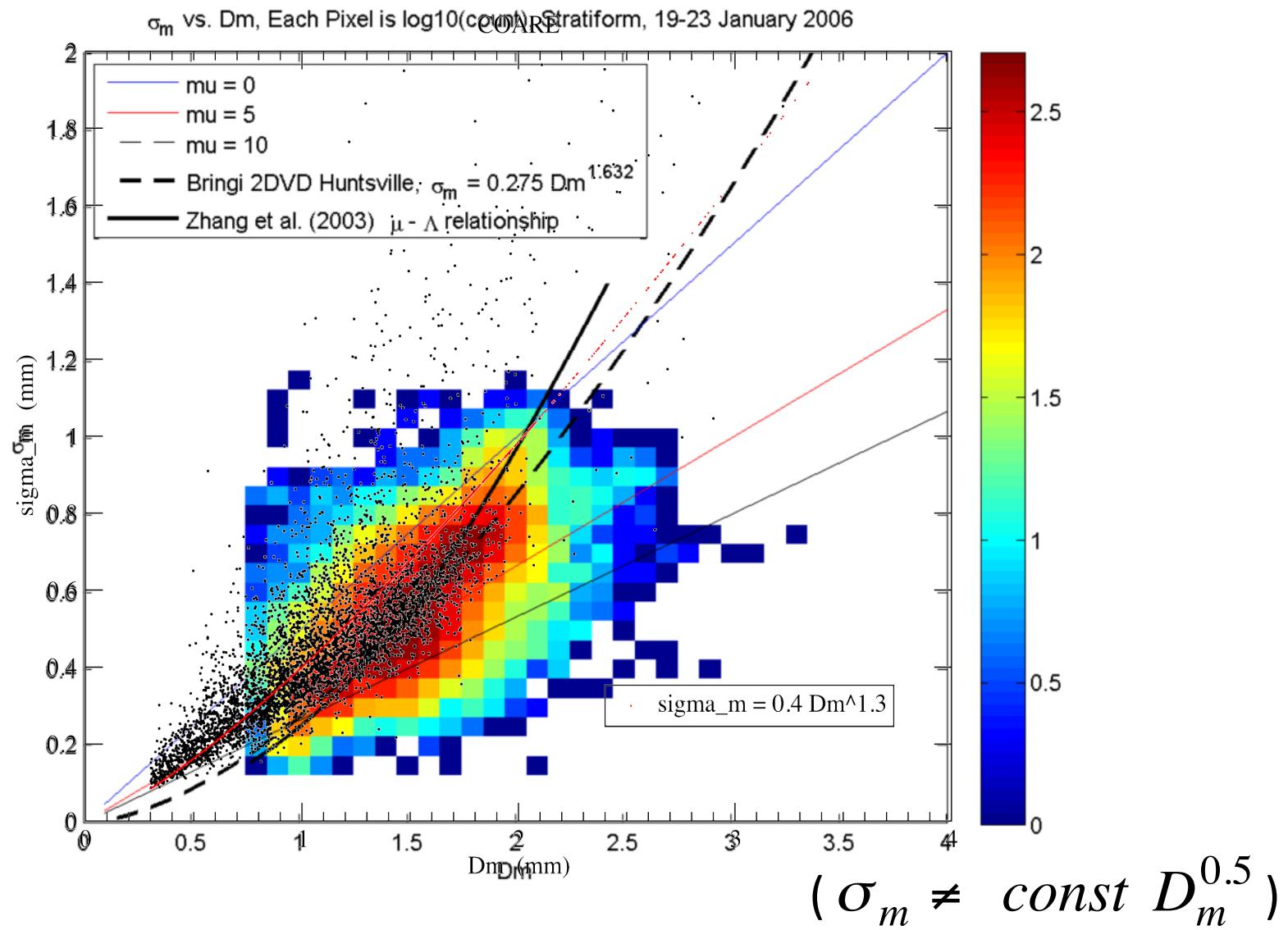
$$\sigma_m = \left( \frac{\int (D - D_m)^2 \ D^3 N(D) \ dD}{\int D^3 N(D) \ dD} \right)^{1/2} = \frac{D_m}{\sqrt{\mu + 4}}$$

The above imply:

$$D_m = \frac{D_m^2 / \sigma_m^2}{\Lambda} \Rightarrow \frac{D_m}{\sigma_m^2} = const \quad , \text{i.e.} \quad \sigma_m = const \ D_m^{0.5}$$

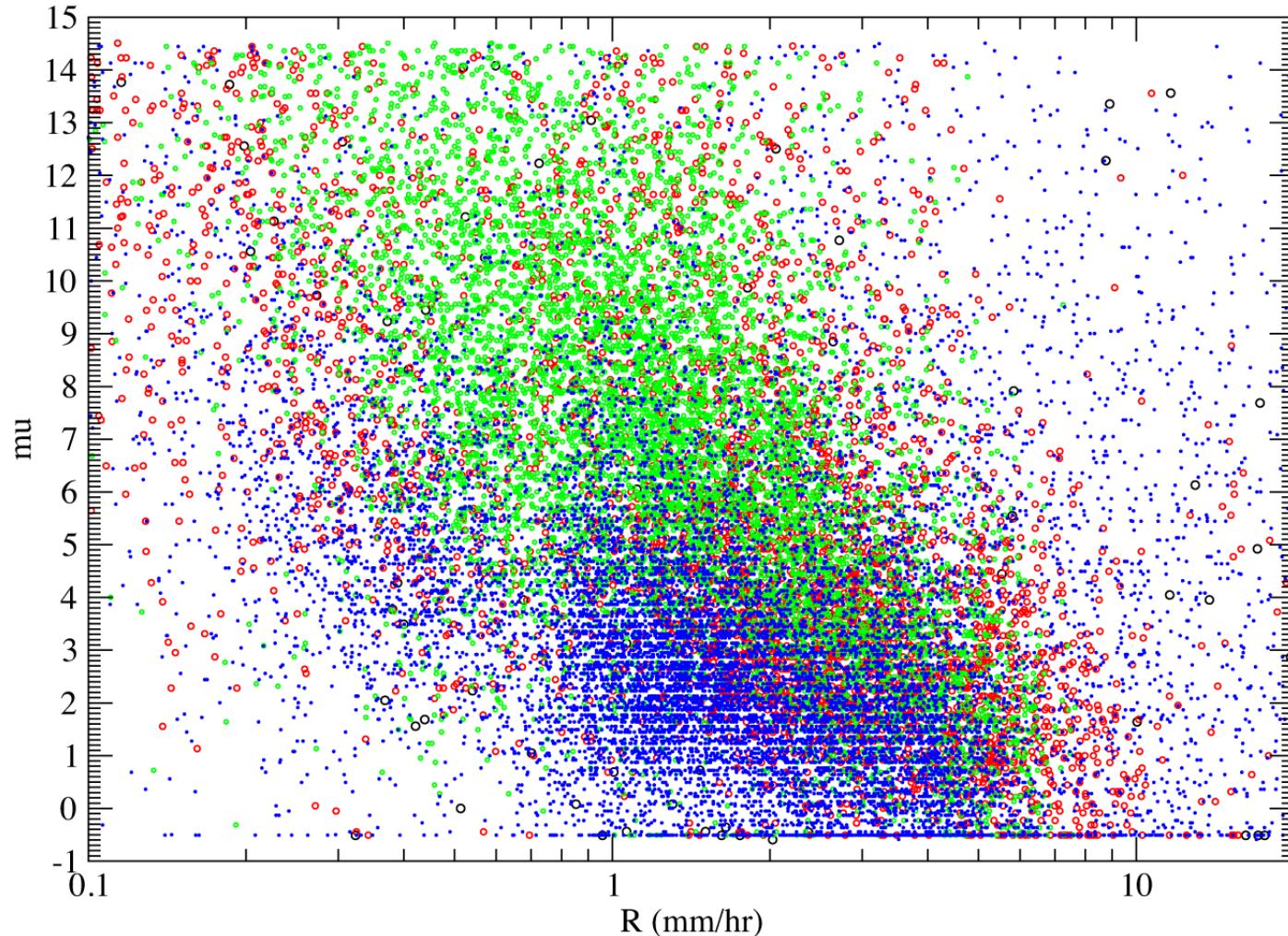
# “unrealistic realism” in hydrometeor size distributions:

⇒ Observations indicate:  $\sigma_m = \text{const} \cdot D_m^{1.5} \pm \text{noise}$



Furthermore,  $\mu$  is neither 0 nor constant (& neither are  $N_0$ ,  $\Lambda$ ):

Darwin profiler, January 19+20 (blue), 22 (red) and 23 (green), 2006



Coup de grâce:

$$\frac{\partial \mu}{\partial t} + V \cdot \nabla \mu = ?$$

$$\frac{\partial \sigma_m}{\partial t} + V \cdot \nabla \sigma_m = ?$$

(and neither parameter is passed on to the radiative transfer ...)

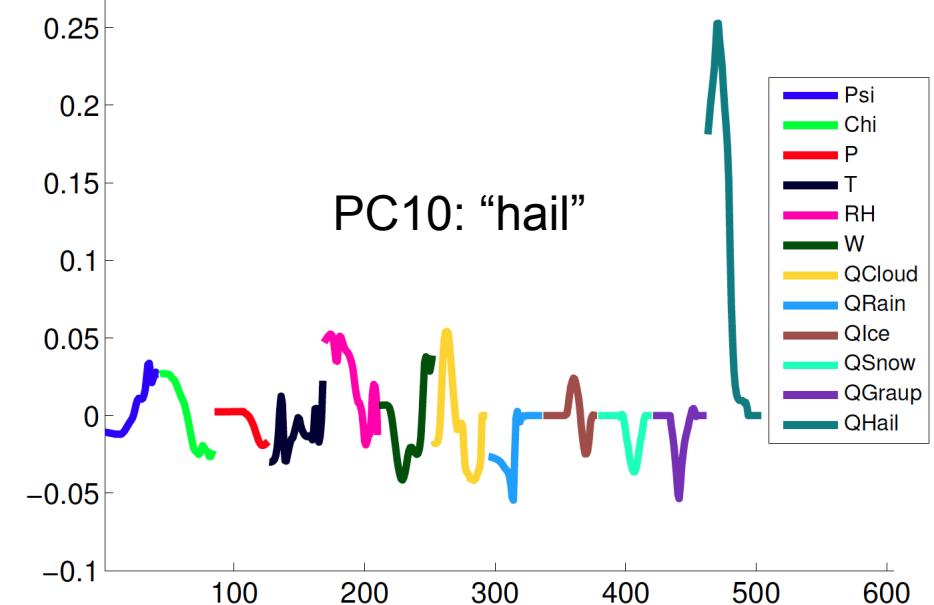
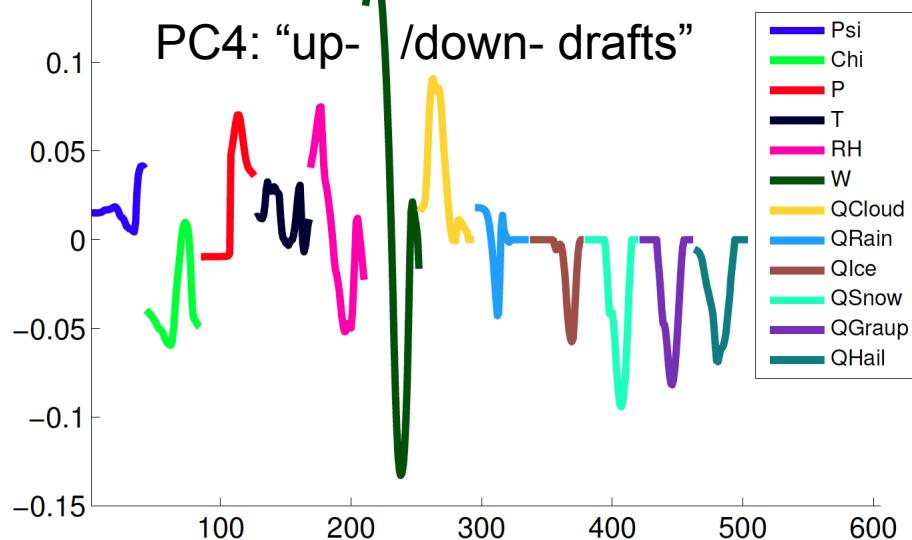
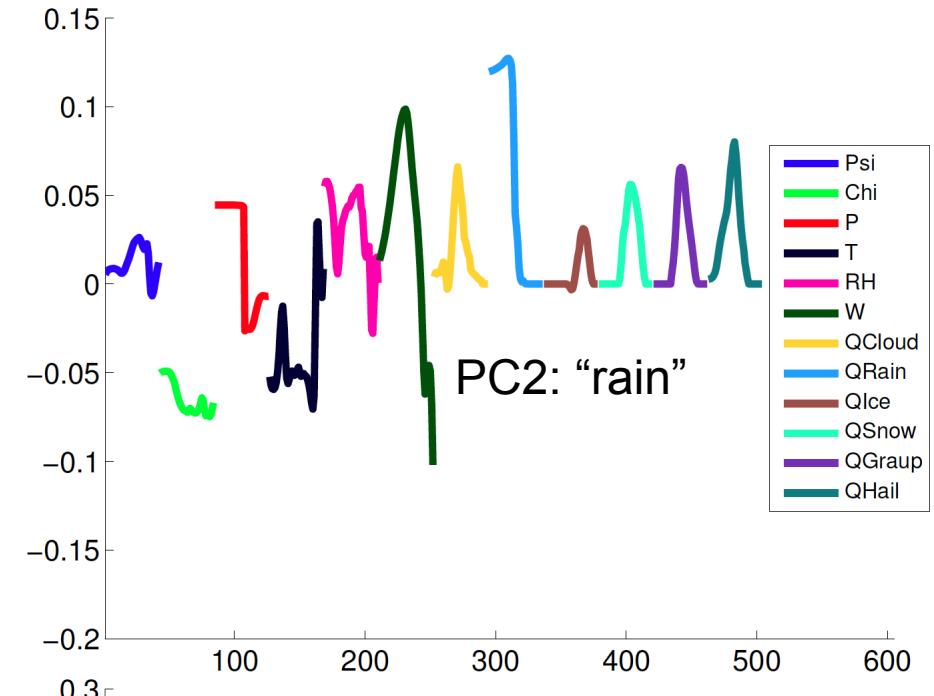
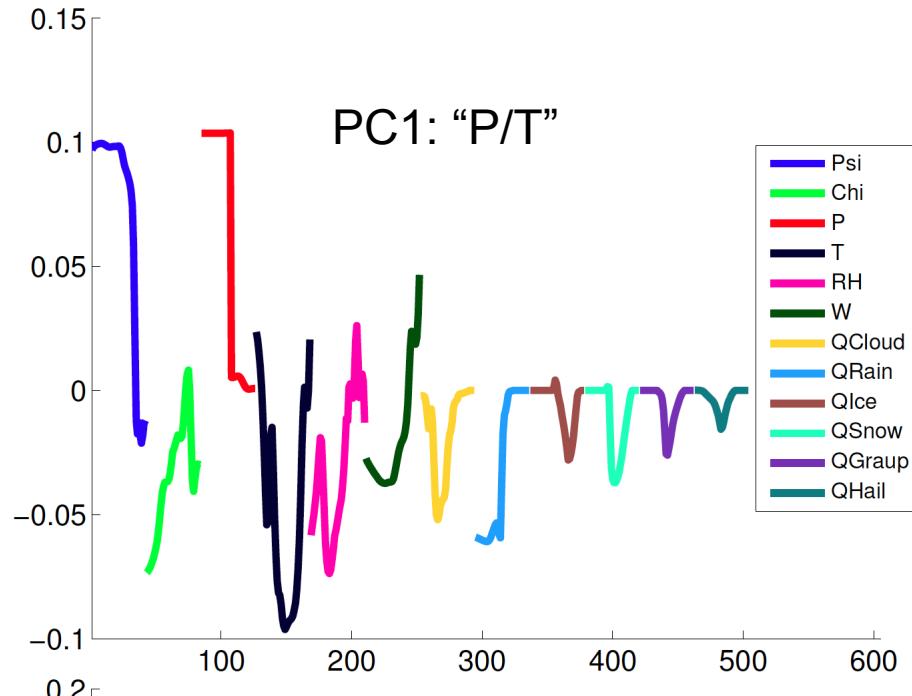
## Alternative:

Change model variables to a new set which

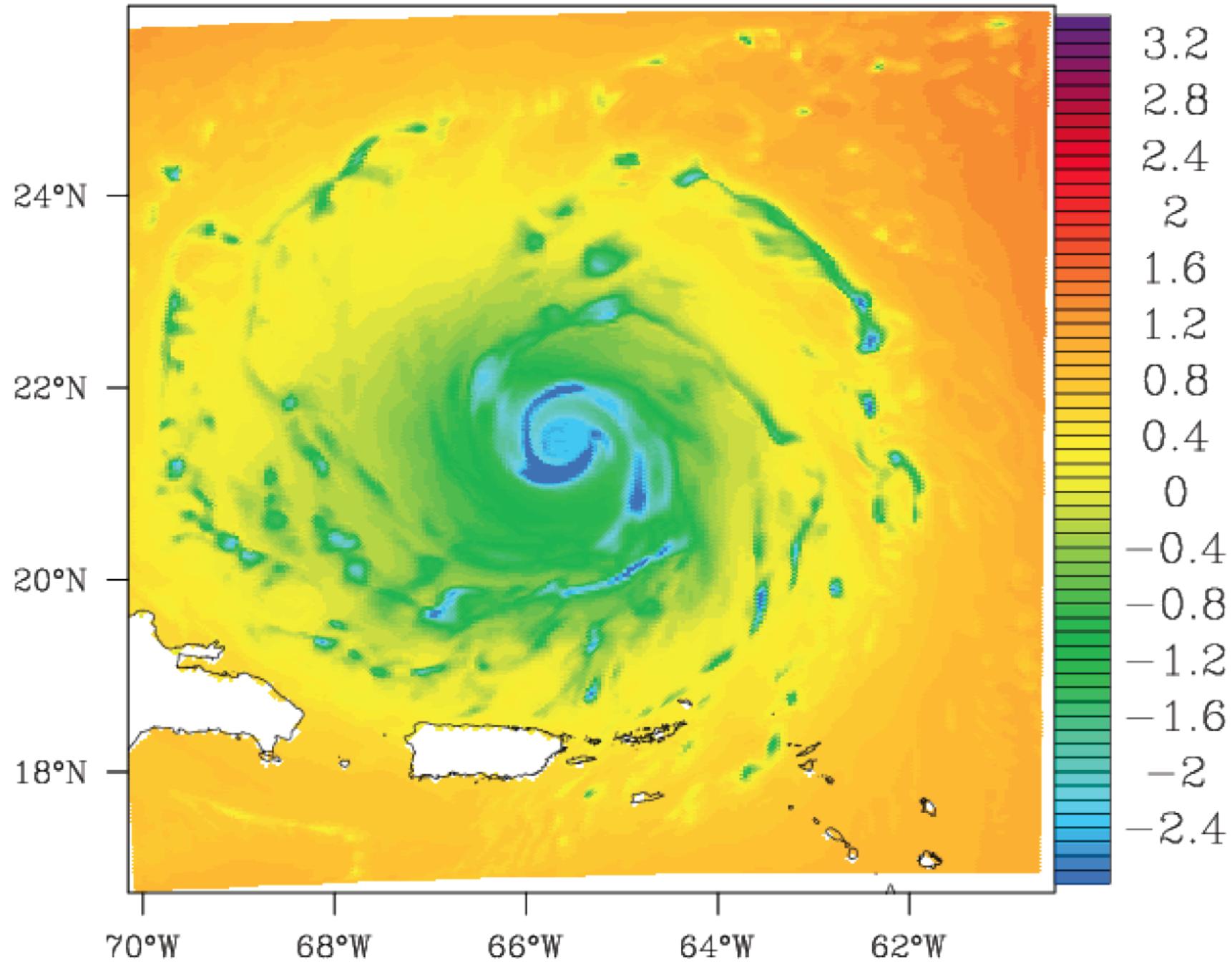
- 1) is small enough so that we don't need  $504 \times 10^3 \times 10^3$  unknowns
- 2) captures most of the variability of the model states
- 3) includes what the observations are sensitive to
- 4) allows the assimilation to adjust all the variables consistently

⇒ vertical principal components of the model variables

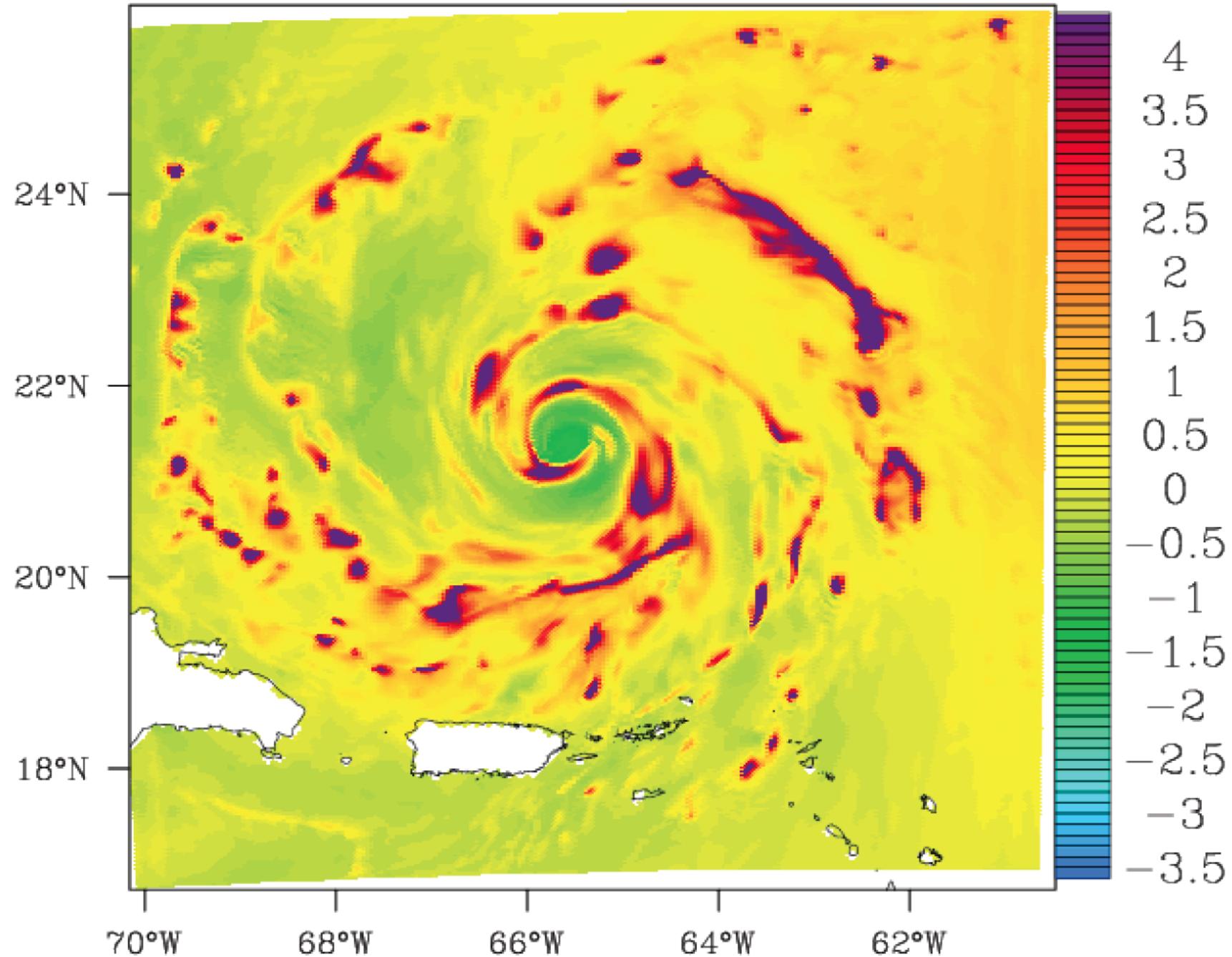
# Analysis of HRD sims of Hurricane Earl: PC coefficients (504 variables)



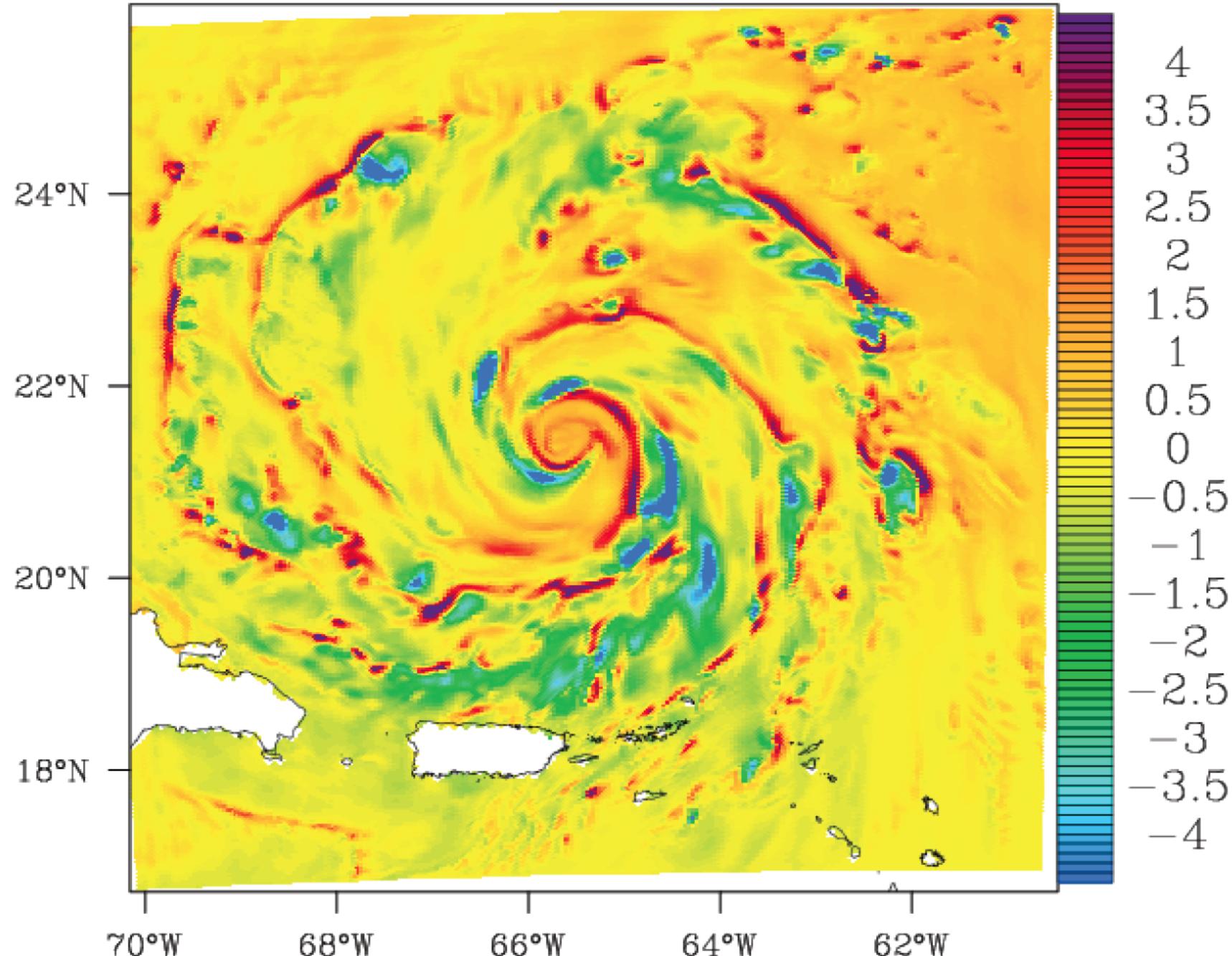
# Analysis of HRD sims of Hurricane Earl: PC 1 ("P/T")



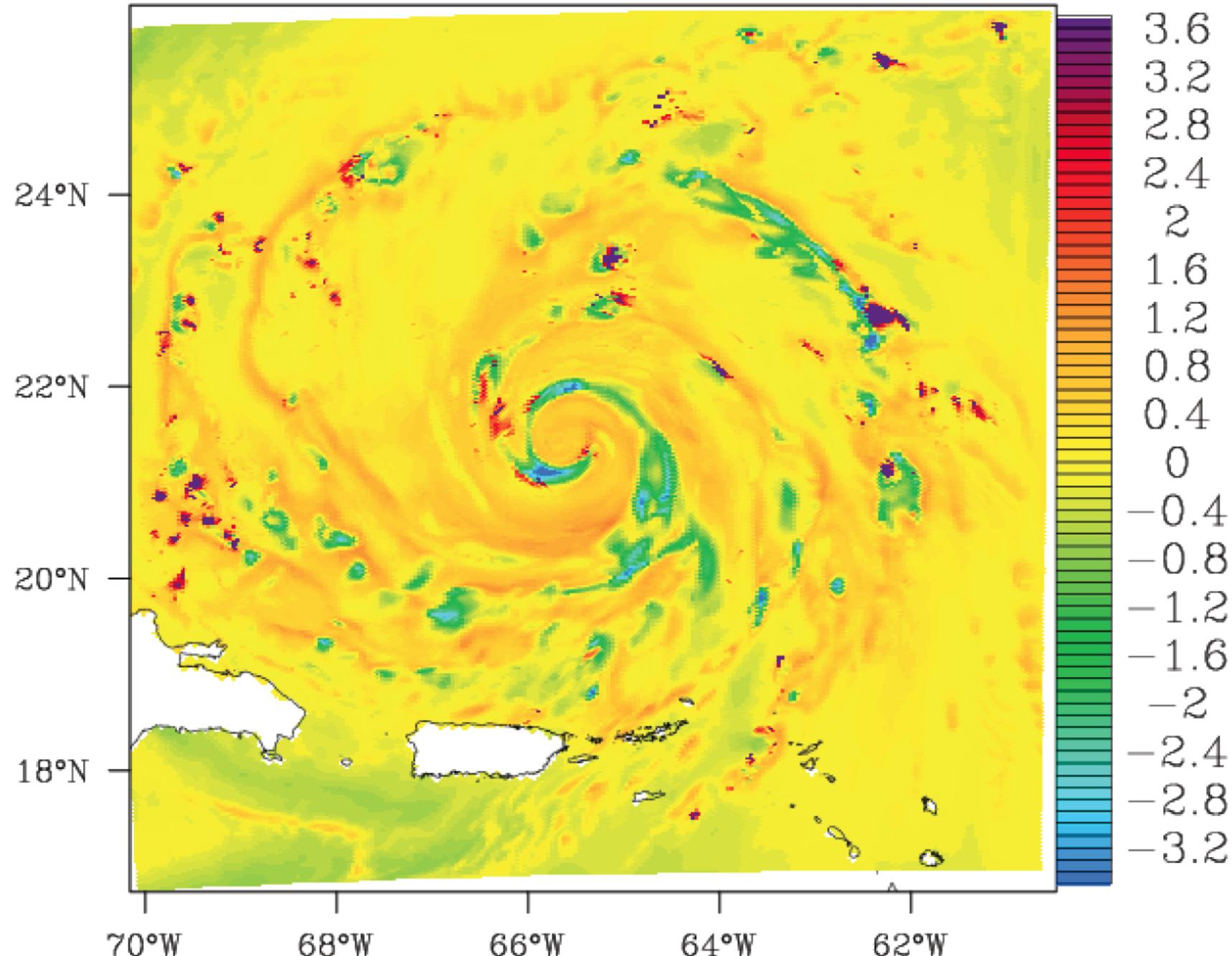
# Analysis of HRD sims of Hurricane Earl: PC 2 ("rain")



# Analysis of HRD sims of Hurricane Earl: PC 4 (" $\omega$ ")



# Analysis of HRD sims of Hurricane Earl: PC 10 ("hail")



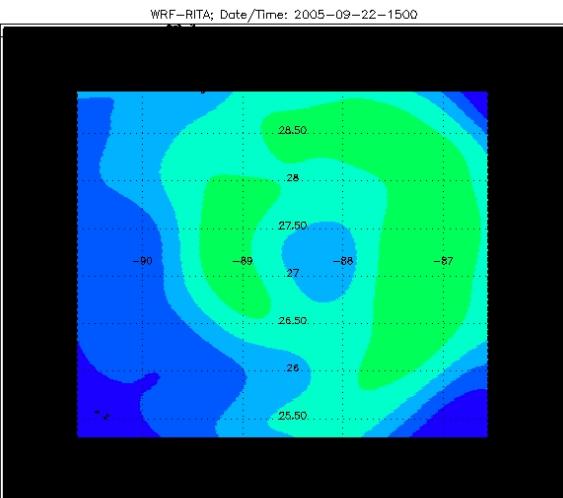
## Observations:

$$\begin{aligned}
 T_b &= \varepsilon(w_s) T_S e^{-\int_0^{\infty} k_{ext}} + \int k_{abs}(h) T(h) e^{-\int_h^{\infty} k_{ext}} dh + \left( \int k_{abs}(h) T(h) e^{-\int_0^h k_{ext}} dh \right) (1 - \varepsilon(w_s)) e^{-\int_0^{\infty} k_{ext}} \\
 &\quad \text{surface } \wedge \quad \text{condensation } \wedge \quad \text{condensation } \vee, \text{ reflected } \wedge \\
 &= \varepsilon(w_s) T_S A(\text{atmosphere}) + B(\text{atmosphere}) + C(\text{atmosphere})(1 - \varepsilon(w_s)) \\
 &= \varepsilon(w_s) \left( T_S A(\text{atmosphere}) + F(\text{atmosphere}) \right) + G(\text{atmosphere}) \\
 &= H(a_1, a_2, \dots, a_7)
 \end{aligned}$$

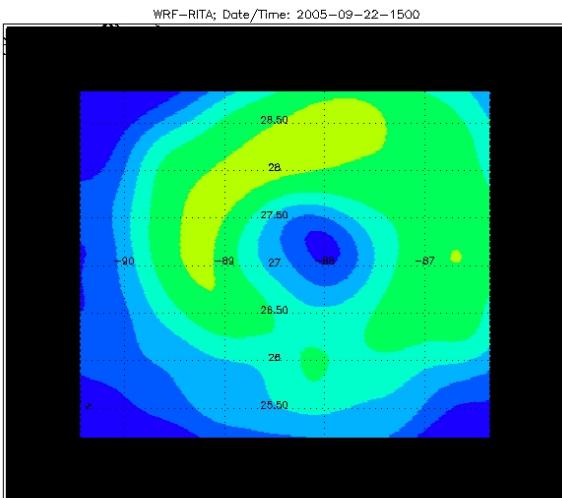
⇒ Our original idea was to assimilate  
 the first two  $T_b$  principal components **PC<sub>1</sub>** and **PC<sub>2</sub>**,  
 expressed in terms of combinations  $a_1, a_2$  and  $a_3$  of the vertical  
 principal components of the model variables with which they are  
 most correlated

# Alternative: forward-calculated Tb (3 different parametrizations)

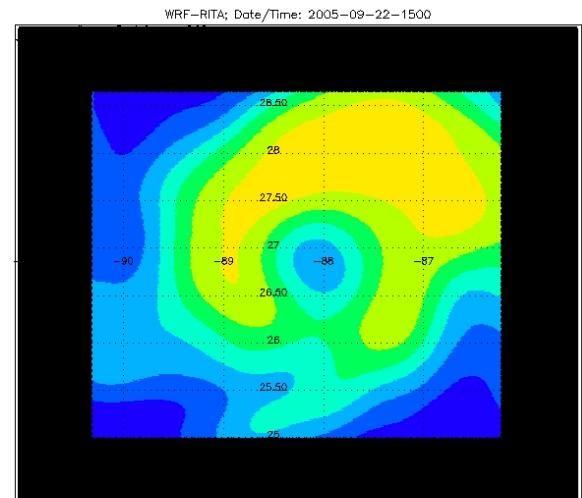
**M3-500.08.04**



**M6-500.08.04**



**M6-300.80.40**



**37H**

100. 140. 170. 190. 210. 220. 230. 240. 250. 260. 270. 280. 290. 295. 300.  
Case:MP3500.08.04; Resolution: WRF\_TMI; TB\_37\_H [K]; Max/Min = 248.79/ 197.86

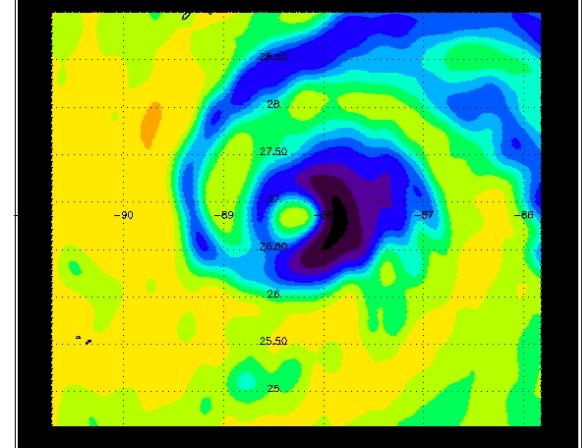
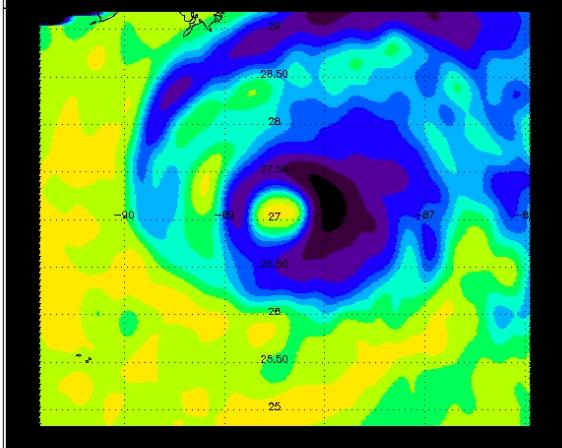
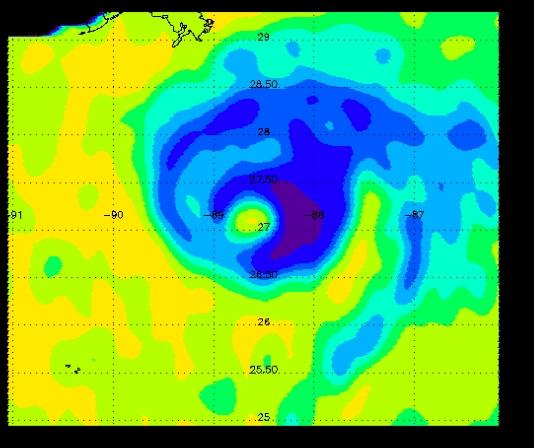
100. 140. 170. 190. 210. 220. 230. 240. 250. 260. 270. 280. 290. 295. 300.  
Case:MP6500.08.04; Resolution: WRF\_TMI; TB\_37\_H [K]; Max/Min = 254.07/ 195.08

100. 140. 170. 190. 210. 220. 230. 240. 250. 260. 270. 280. 290. 295. 300.  
Case:MP6300.80.40; Resolution: WRF\_TMI; TB\_37\_H [K]; Max/Min = 268.81/ 196.49

WRF-RITA; Date/Time: 2005-09-22-1500

WRF-RITA; Date/Time: 2005-09-22-1500

WRF-RITA; Date/Time: 2005-09-22-1500



**85H**

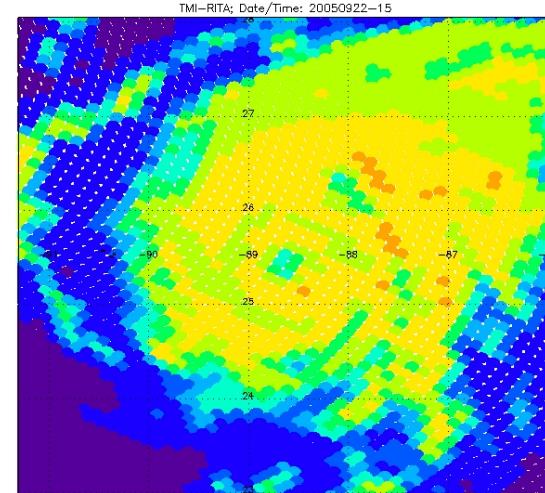
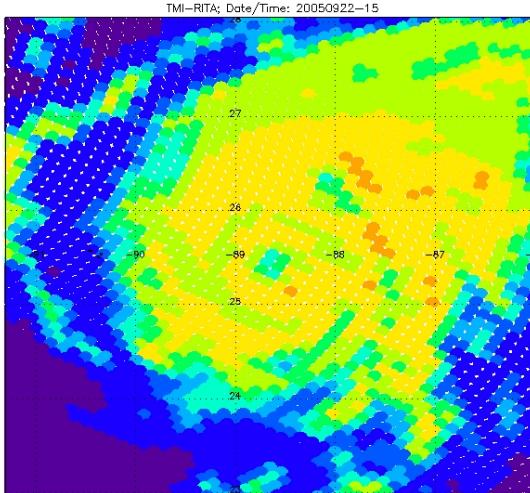
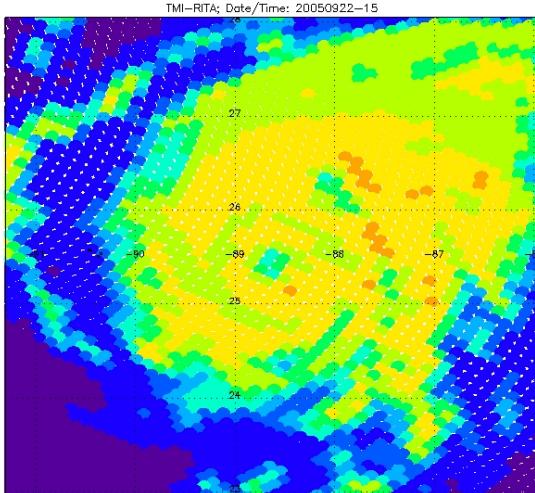
100. 140. 170. 190. 210. 220. 230. 240. 250. 260. 270. 280. 290. 295. 300.  
Case:MP3500.08.04; Resolution: WRF\_TMI; TB\_85\_H [K]; Max/Min = 266.40/\*\*\*\*\*

100. 140. 170. 190. 210. 220. 230. 240. 250. 260. 270. 280. 290. 295. 300.  
Case:MP6500.08.04; Resolution: WRF\_TMI; TB\_85\_H [K]; Max/Min = 265.47/\*\*\*\*\*

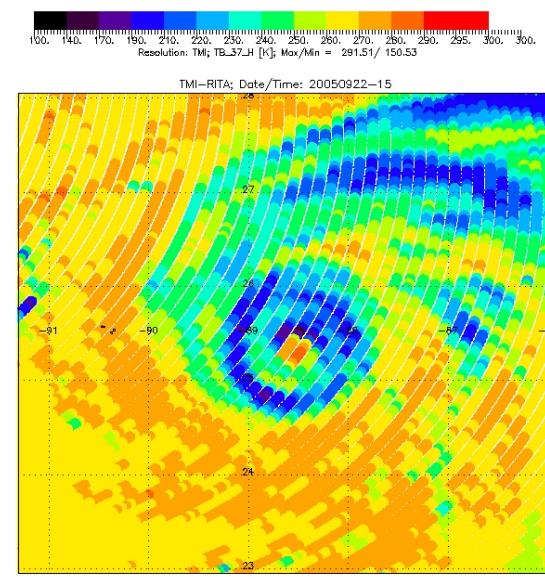
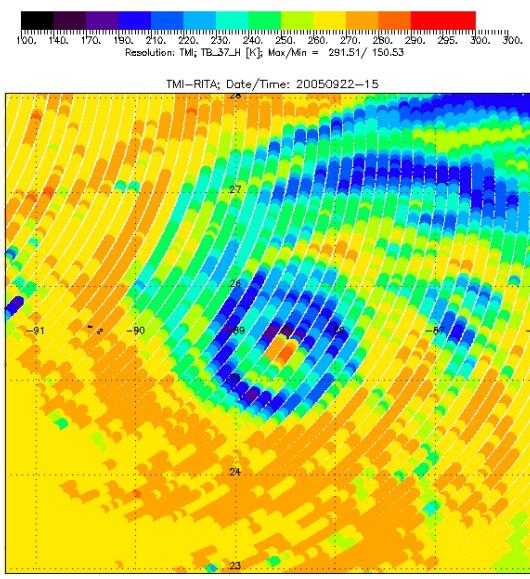
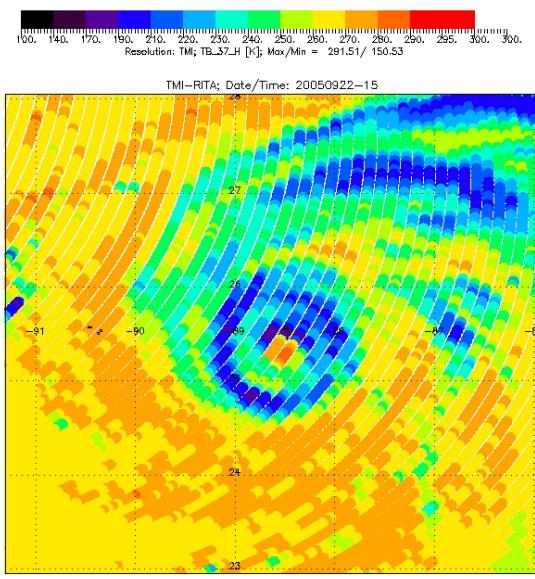
100. 140. 170. 190. 210. 220. 230. 240. 250. 260. 270. 280. 290. 295. 300.  
Case:MP6300.80.40; Resolution: WRF\_TMI; TB\_85\_H [K]; Max/Min = 271.74/ 129.85

# Alternative: TMI $T_b$ (observed)

Quite Different!



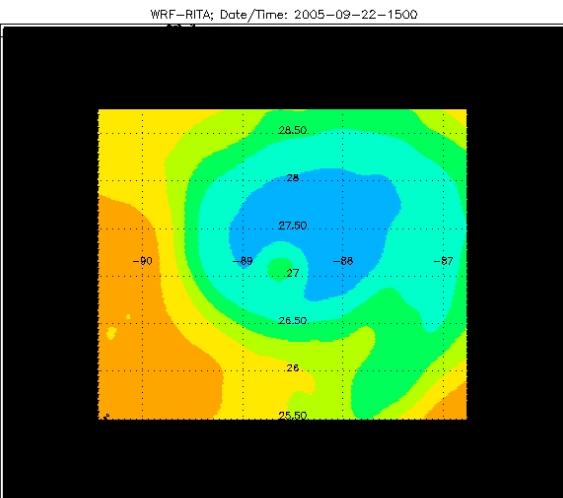
37H



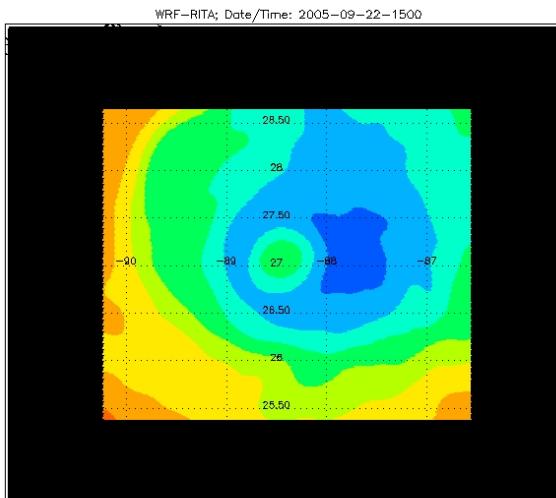
85H

# Alternative: PCs of forward-calculated Tb (same 3 parametrizations)

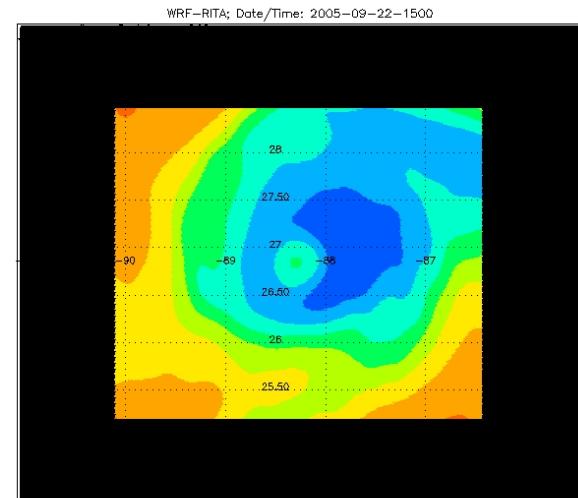
**M3-500.08.04**



**M6-500.08.04**

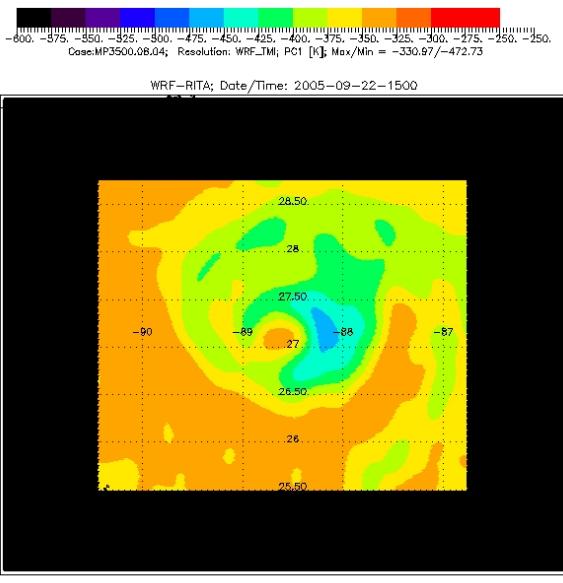


**M6-300.80.40**

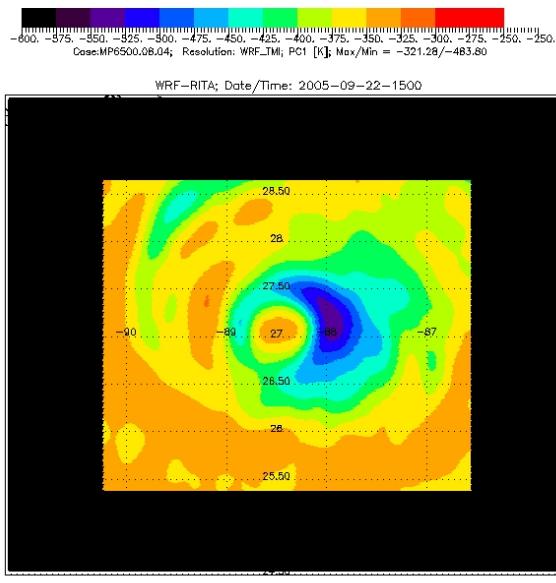


**PC<sub>1</sub>**

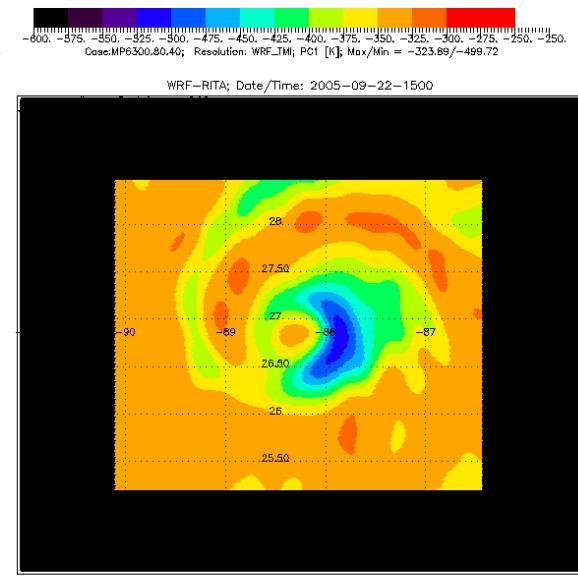
WRF-RITA; Date/Time: 2005-09-22-1500



WRF-RITA; Date/Time: 2005-09-22-1500



WRF-RITA; Date/Time: 2005-09-22-1500



**PC<sub>2</sub>**

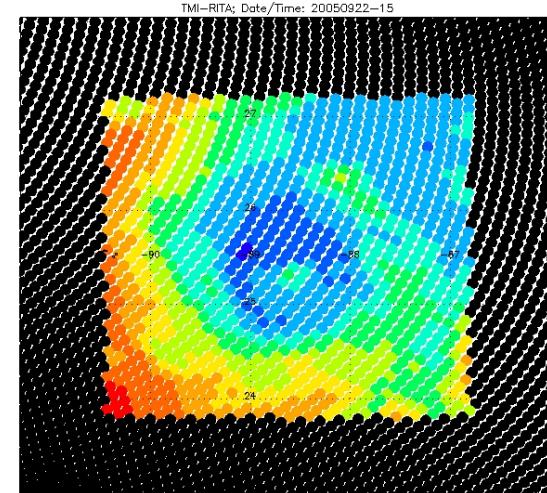
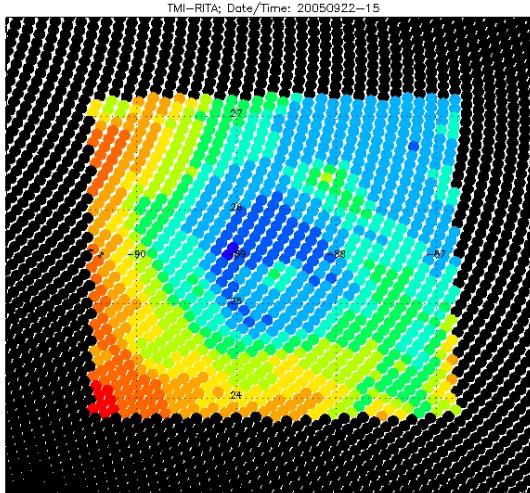
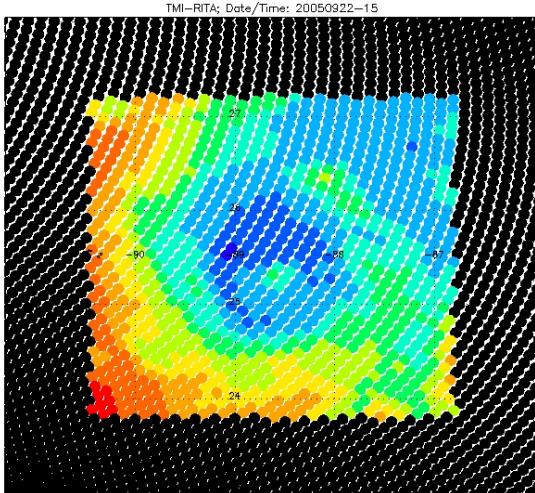
250. 275. 300. 325. 350. 375. 400. 425. 450. 475. 500. 525. 550. 575. 600. Case:MP3500.08.04; Resolution: WRF\_TMI; PC2 [K]; Max/Min = 522.39/389.58

250. 275. 300. 325. 350. 375. 400. 425. 450. 475. 500. 525. 550. 575. 600. Case:MP6500.08.04; Resolution: WRF\_TMI; PC2 [K]; Max/Min = 525.57/308.96

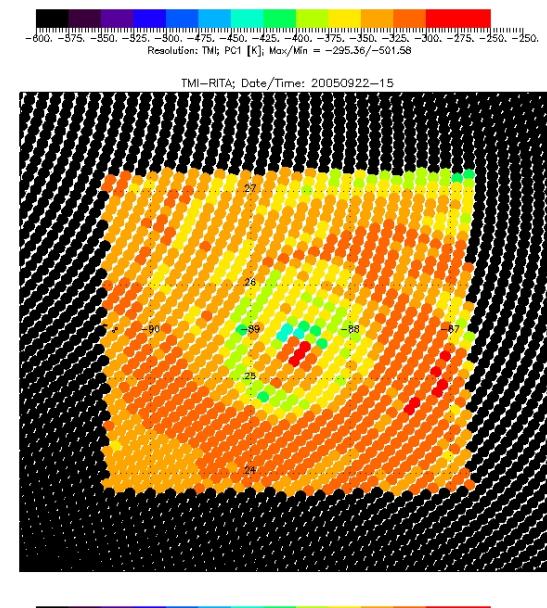
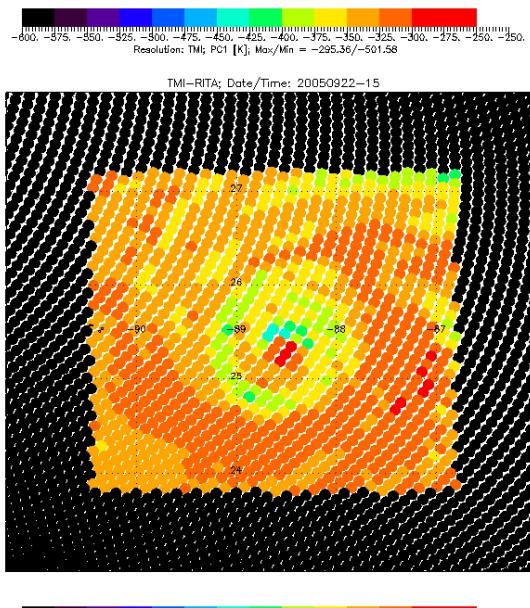
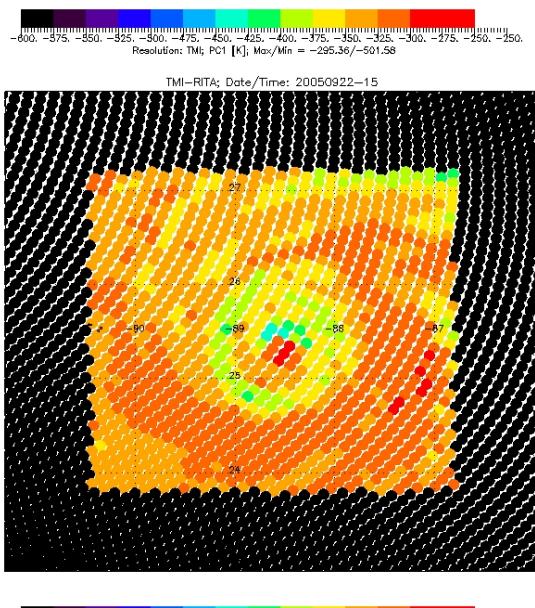
250. 275. 300. 325. 350. 375. 400. 425. 450. 475. 500. 525. 550. 575. 600. Case:MP6300.80.40; Resolution: WRF\_TMI; PC2 [K]; Max/Min = 530.69/337.90

# Alternative: PCs of TMI $T_b$ (observed)

PC<sub>1</sub> not so Different!



PC<sub>1</sub>



PC<sub>2</sub>

## Alternative:

⇒ which combination(s) of  $T_b$  are “optimal” to assimilate?

linear answer:

-0.12  $T_{10.7V}$  -0.41  $T_{10.7H}$  -0.05  $T_{19.3V}$  -0.36  $T_{19.3H}$  +0.05  $T_{23V}$  +0.26  $T_{37V}$  -0.19  $T_{37H}$  +0.61  $T_{89V}$  +0.46  $T_{89H}$   
(atmospheric scattering index)

-0.21  $T_{10.7V}$  +0.16  $T_{10.7H}$  +0.18  $T_{19.3V}$  +0.36  $T_{19.3H}$  +0.19  $T_{23V}$  +0.66  $T_{37V}$  +0.48  $T_{37H}$  +0.25  $T_{89V}$  +0.09  $T_{89H}$   
(atmospheric emission index)

+0.12  $T_{10.7V}$  +0.12  $T_{10.7H}$  +0.12  $T_{19.3V}$  +0.26  $T_{19.3H}$  +0.70  $T_{23V}$  +0.05  $T_{37V}$  +0.43  $T_{37H}$  -0.35  $T_{89V}$  +0.29  $T_{89H}$   
(surface wind + vapor)

⇒ More complete answer + observation operator + +  
in the next installment